

Modeling and Uncertainty Quantification for Airfoil Icing

Anthony DeGennaro
Clarence W. Rowley III
Luigi Martinelli
Princeton University

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Outline

1 Motivation/Background

2 Heuristic UQ

3 Data-Based UQ

4 Computational UQ



Topic

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2 Heuristic UQ

3 Data-Based UQ

4 Computational UQ



Introduction

Wing icing deteriorates airfoil aerodynamics

- Leading edge flow separation bubble
- Lower lift, higher drag
- Unpredictable stall

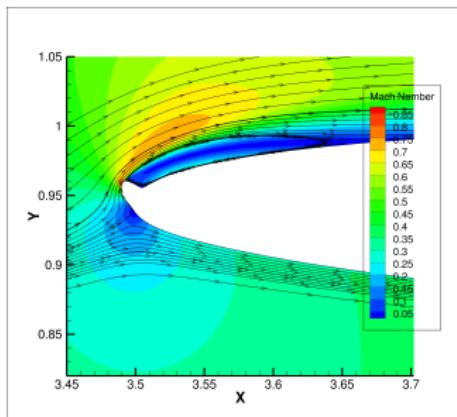


Figure: Leading edge horn separation



Introduction

Significant ice shape variation, sensitivity to physical parameters¹

- Complex physics (aero-thermodynamics, macro/micro scale physics)
- Uncertainty in physical parameters

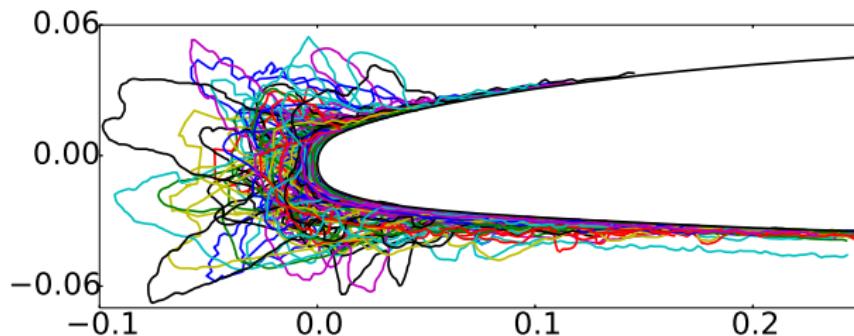


Figure: Wind tunnel experimental ice shapes

¹ Addy, H.E. *Ice Accretions and Icing Effects for Modern Airfoils*. NASA TR-2000-210031.

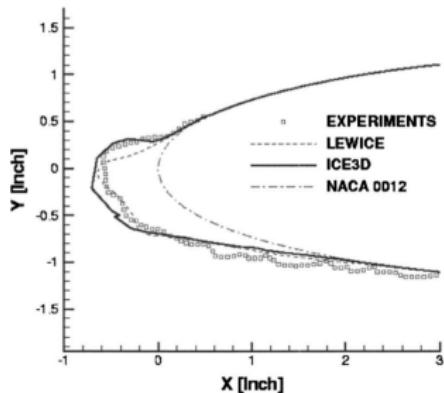




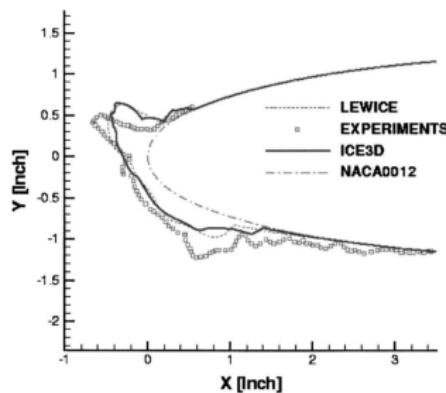
Introduction

Different types of ice accretion²

- “Horns”, “ridges”, “lobster tails” refer to shape
- “Glaze”, “rime” refer to icing thermodynamics



(a) Rime Ice



(b) Horn Ice

²Beaugendre et. al. *Development of a Second Generation In-Flight Simulation Code*. J. Fluids Engineering, 2006.



Introduction

Research Goals

- Apply uncertainty quantification techniques to explore statistical effects of uncertain icing parameters on ice shape and aerodynamics
 - Polynomial chaos expansions (PCE)
 - Tensor/sparse grid collocation sampling
 - 2D steady-state RANS solver for aerodynamic assessment
- Build ice shape model from data
 - Aggregate ice shape database
 - Cluster shapes using spectral graph partitioning
 - Model shape variation using Proper Orthogonal Decomposition (POD)
- Quantify effects of physical uncertainties in aero-thermodynamics
 - Build a computational ice-accretion code
 - UQ on governing parameters (LWC, temperature, accretion time, etc.)



Introduction

Outline of Research

- Heuristic UQ
 - Ice shape scaling parameters
 - Verify PCE techniques against Monte Carlo simulations
- Data-based UQ
 - Build ice shape model from data
 - Clustering + POD
- Computational UQ
 - Build computational ice accretion code
 - Droplet impingement + thermodynamic PDE solvers



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Canonical Ice Shapes

- Basic scalings/translations³

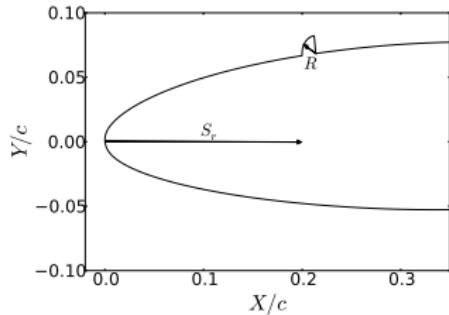


Figure: Ridge Parameterization

- Ridge radius
- Ridge position

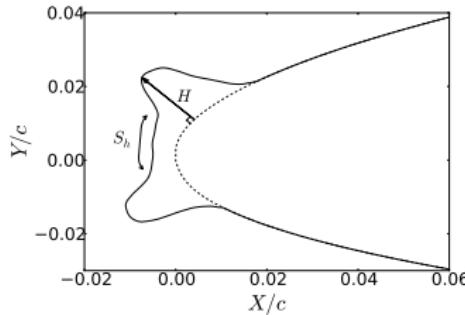


Figure: Horn Parameterization

- Horn height
- Horn separation

³DeGennaro A., Rowley C.W., and Martinelli, L. *Uncertainty Quantification for Airfoil Icing using Polynomial Chaos Expansions*. Journal of Aircraft, 2015.



Application to Icing UQ

- We wish to apply a fast and accurate method for quantifying uncertainty in the aerodynamics of these ice shapes
- Choose to use polynomial chaos expansions (PCE)
 - Fast compared to Monte Carlo
 - Explicit surrogate
 - Easy statistical sampling
 - Can compute sensitivities, analysis of variance
- We will compute UQ results for horn and ridge problems using PCE, and verify them against high-resolution Monte Carlo simulations



Ridge Study

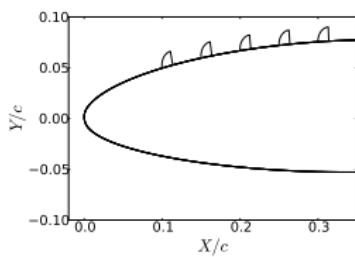
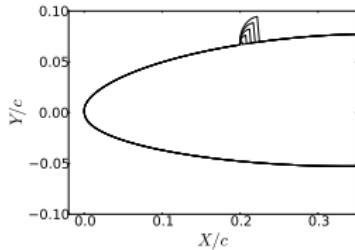


Figure: Ridge Variations

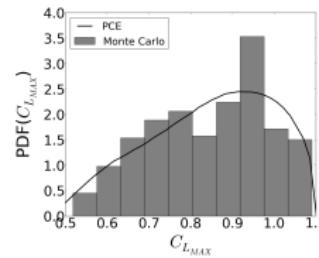
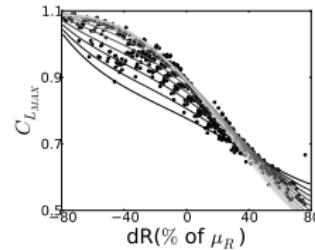


Figure: Lift Statistics

- Ridge parameters normally distributed about mean
- Performance degrades with larger size, closer to L.E.



Horn Study

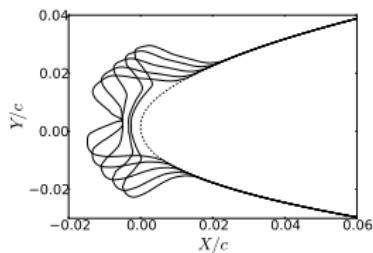
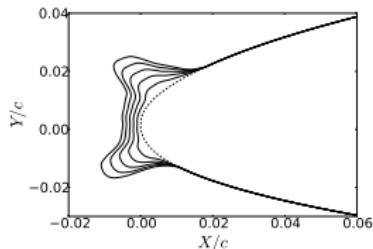


Figure: Ridge Variations

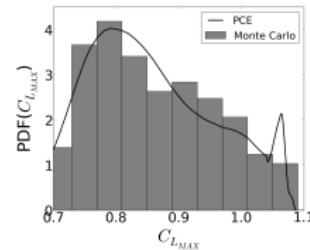
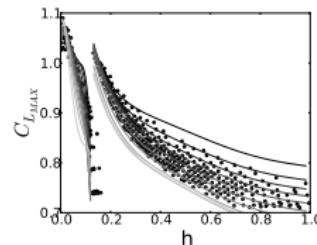


Figure: Lift Statistics

- Horn separation normally distributed
- Horn height distributed as a half-Gaussian (mean = clean airfoil)
- Performance degrades with larger horn size and separation



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Dataset

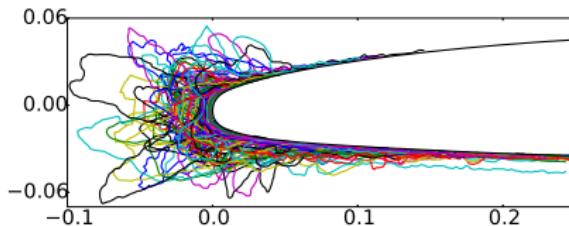


Figure: Wind tunnel experimental ice shapes

- Dataset consists of 145 experimental ice shapes
- Obtained in icing wind tunnel at NASA Glenn¹
- Representative of a wide variety of icing conditions (temperature, LWC, accretion time, etc.)



Dataset

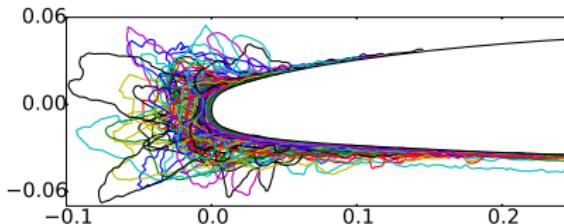


Figure: Wind tunnel experimental ice shapes

Goals:

- Make the ice shapes studied in UQ better reflect observed data
- Build low-dimensional models to describe complex data
- Develop empirical ice classification scheme

Approach:

- Cluster ice shapes using spectral graph partitioning
- Build low-dimensional model using POD
- Perform parametric UQ on resulting parameter space



Distance/Similarity Metric

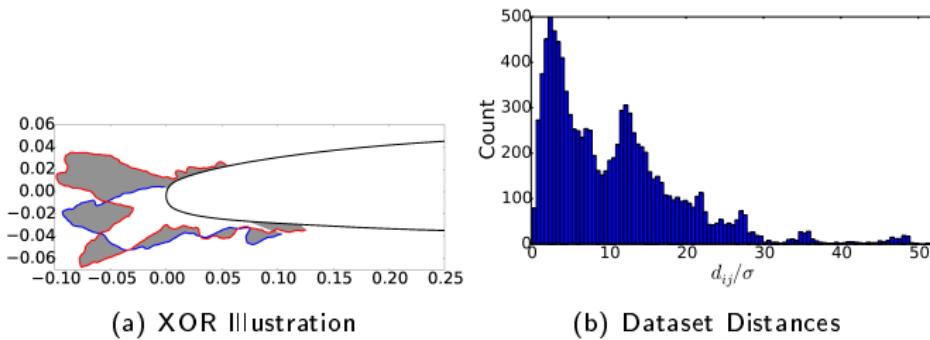


Figure: XOR distance metric

- Overlay dataset with a 2D Cartesian grid
- Assign value of 1 to gridpoint if it is on the ice, 0 otherwise
- Pick σ based on observed peaks in data distances
- Truncate w_{ij} after $d_{ij} > 3\sigma$

$$w_{ij} = \exp\left(-\frac{1}{2} \frac{d_{ij}^2}{\sigma^2}\right) \quad w_{ij} = \sum_k [\text{XOR}(x_i, x_j)]_k$$



Spectral Graph Partitioning

Goal: cluster shapes based upon similarity metric

Methodology: view database as an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$

- Vertices \mathcal{V} are ice shapes
- Edges \mathcal{E} are similarities between ice shapes
- Find “best” partition of $\mathcal{G}(\mathcal{V}, \mathcal{E})$ into subsets A and B

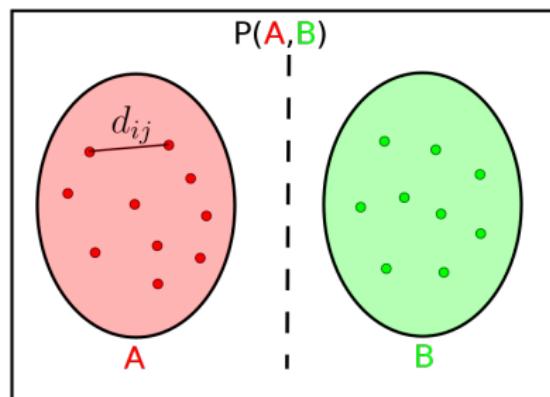


Figure: Graph partition illustration



Spectral Graph Partitioning

Approach:⁴

- Calculate graph Laplacian using similarity metric
 - Similarity matrix: $W = w(i, j)$
 - Degree matrix: $D = \text{diag}(d_k) , d_k = \sum_{j=1}^N w(v_k, v_j) , k = 1 \dots N$
 - Laplacian matrix: $L = D - W$
- Eigenvectors with zero eigenvalue identify disconnected subsets
 - E.g., $L\mathbf{1} = \mathbf{0} \iff$ entire graph is disconnected
- First nonzero eigenvector (Fiedler vector) identifies optimal partition of connected vertices within subsets
 - Approximates solution of *average cut* formulation:
$$P(A, B) = \min_{A \in \mathcal{V}} \left\{ \frac{\text{cut}(A, B)}{|A|} + \frac{\text{cut}(A, B)}{|B|} \right\}$$

 $|A| = \text{Number of vertices in } A$
 $\text{cut}(A, B) = \sum_{u \in A, v \in B} w(u, v)$
 - Eigenvectors close to zero similarly identify good partitions

⁴Shi & Malik. *Normalized Cuts and Image Segmentation*, 2000.



Spectral Graph Partitioning



Figure: Toy example

$$L = \begin{bmatrix} 1 & -0.9 & -0.1 & 0 & 0 & 0 & 0 \\ -0.9 & 1 & -0.1 & 0 & 0 & 0 & 0 \\ -0.1 & -0.1 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -0.5 & -0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 1 & 0 & -0.5 \\ 0 & 0 & 0 & -0.5 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & -0.5 & 1 \end{bmatrix}$$



Spectral Graph Partitioning

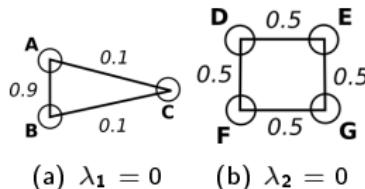


Figure: Disconnected subsets

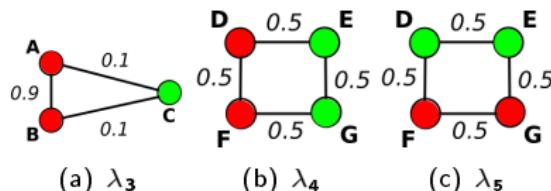


Figure: Clustering within subsets

- Two zero eigenvalues, corresponding to two clusters
- Eigenvalues close to zero give good partitions within clusters



Graph Laplacian

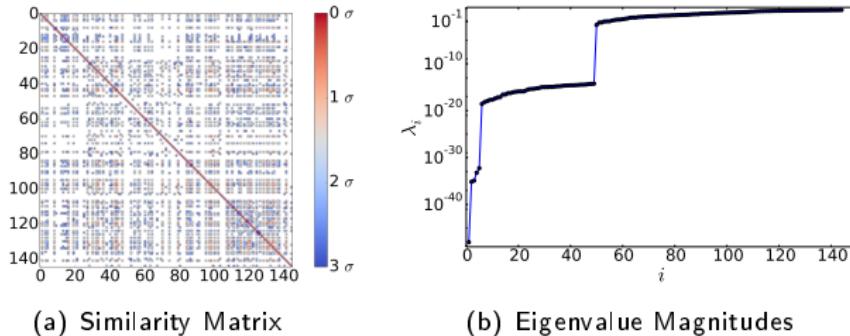
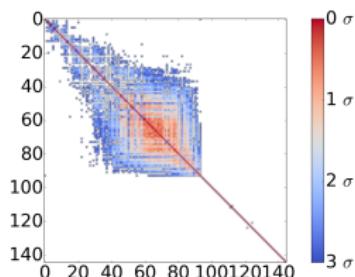


Figure: Laplacian visualization and eigenvalues

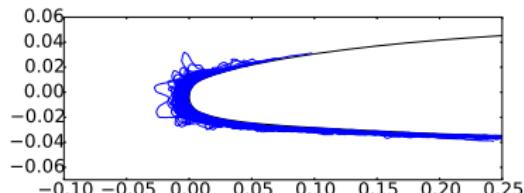
- Many zero eigenvalues because many of the dataset elements are completely unconnected from each other



Clusters



(a) Similarity Matrix



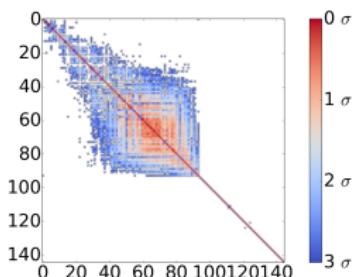
(b) Ice shapes

Figure: $\lambda = 0$

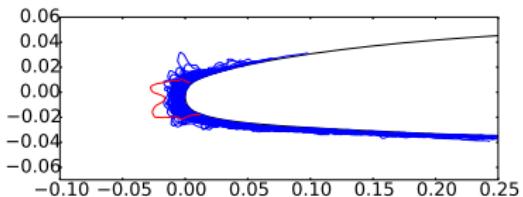
- Unconnected cluster represents smaller and less “extreme” shapes



Clusters



(a) Similarity Matrix



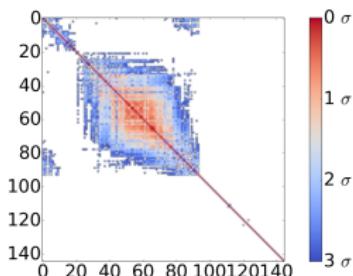
(b) Ice shapes

Figure: Fiedler vector

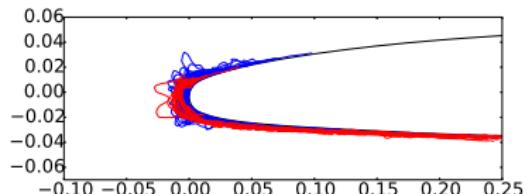
- Fiedler vector partitions off single most dissimilar member



Clusters



(a) Similarity Matrix



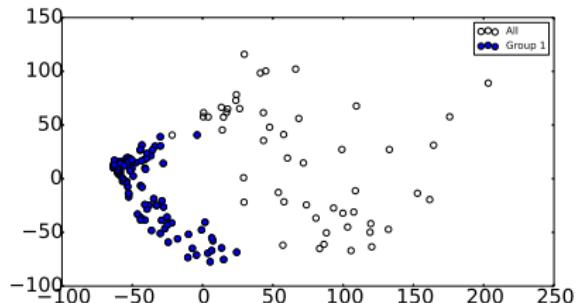
(b) Ice shapes

Figure: Next smallest eigenvector

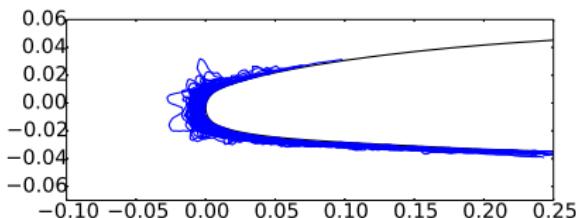
- Next smallest eigenvector separates horn and rime accretion



POD Coordinates



(a) POD coordinates

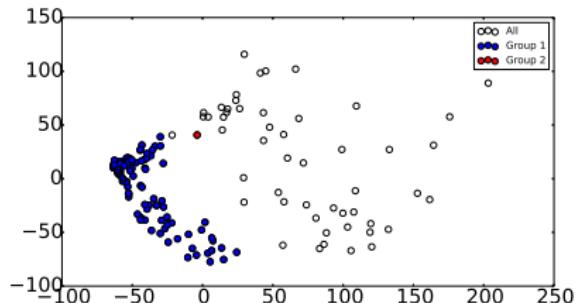


(b) Ice shapes

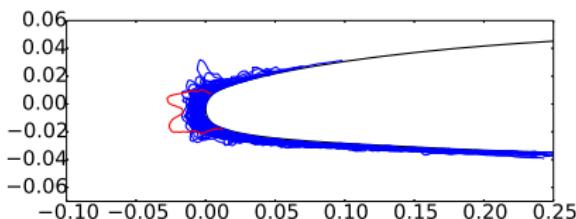
Figure: $\lambda = 0$



POD Coordinates



(a) POD coordinates

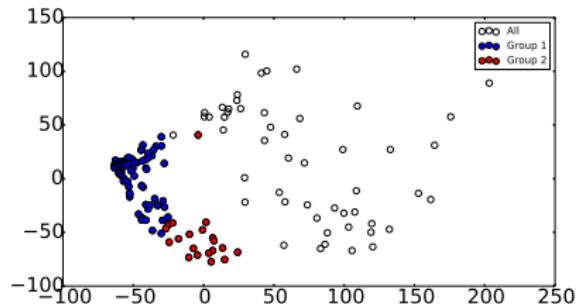


(b) Ice shapes

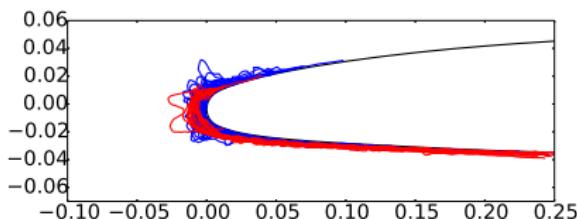
Figure: Fiedler vector



POD Coordinates



(a) POD coordinates



(b) Ice shapes

Figure: Next smallest eigenvector



Cluster Modeling

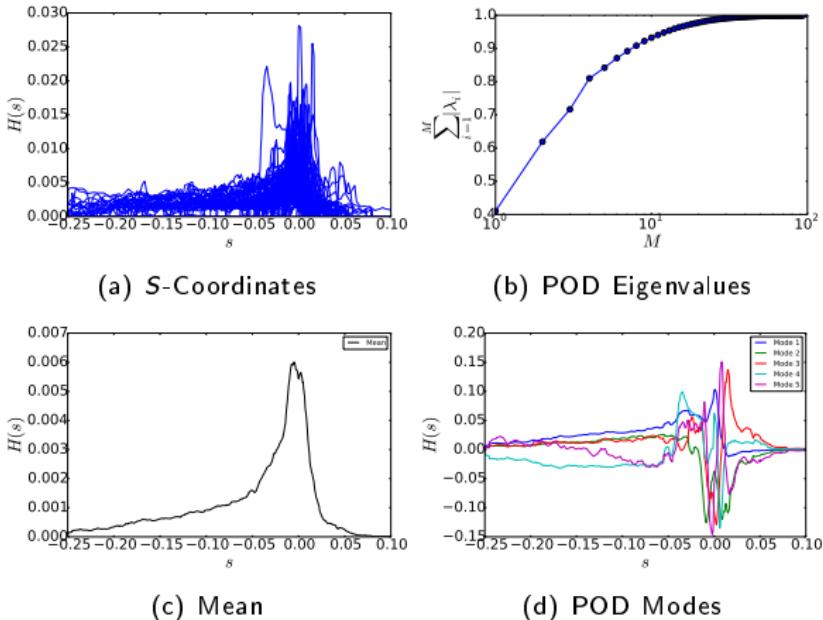


Figure: POD of ice shape cluster

Goal: build a low-dimensional model of ice shape cluster for UQ



Cluster Modeling

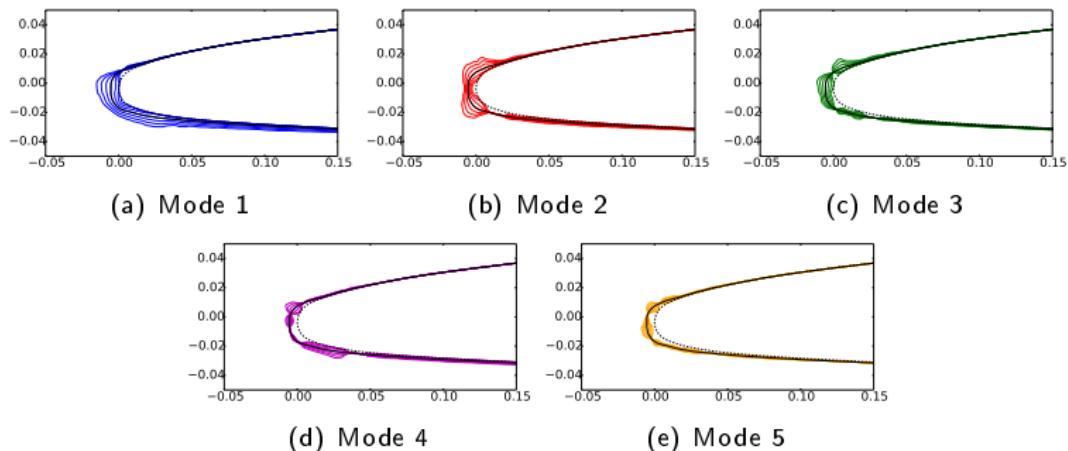


Figure: Ice model modes

Variations shown about dataset mean ($\pm 3\sigma$)



Parameter Space

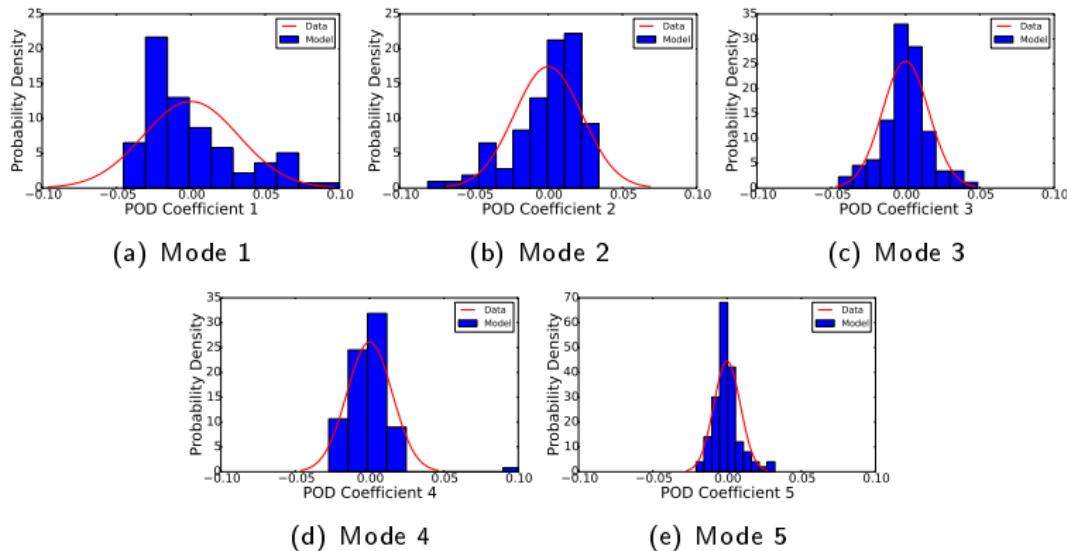


Figure: Mode statistics (data and fit)

- Fit a normal distribution to dataset statistics
- 5-dimensional UQ study with all Gaussian variables



Output Statistics

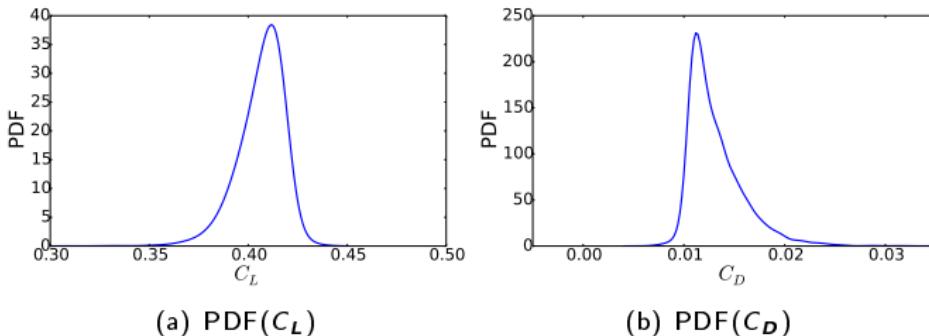


Figure: Output PDFs for lift and drag

Setup

- Business jet clean airfoil¹, $\alpha = 3^\circ$, $Re = 7.5 \times 10^6$
- FLO103 code (2D steady-state RANS solver)
- Adaptive sparse grid collocation for PCE, implemented with DAKOTA

Results

- PC surrogate converged using 487 solver evaluations



Global Extrema

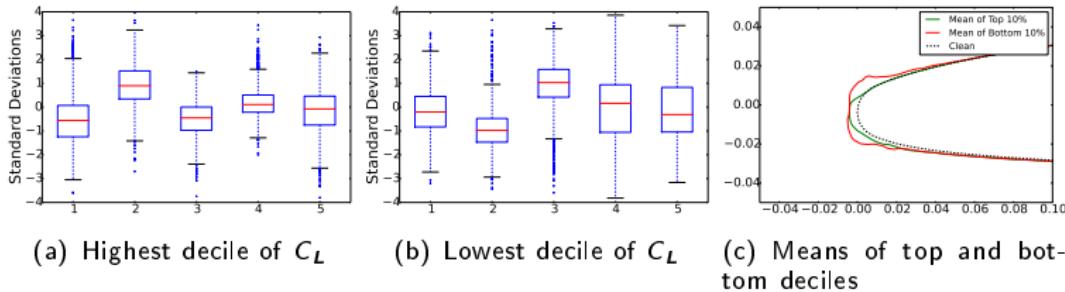


Figure: Global extrema visualized

- **Good:** low accretion, smooth, conforms to airfoil surface
- **Bad:** high accretion, horns, protrude out as flow obstacles



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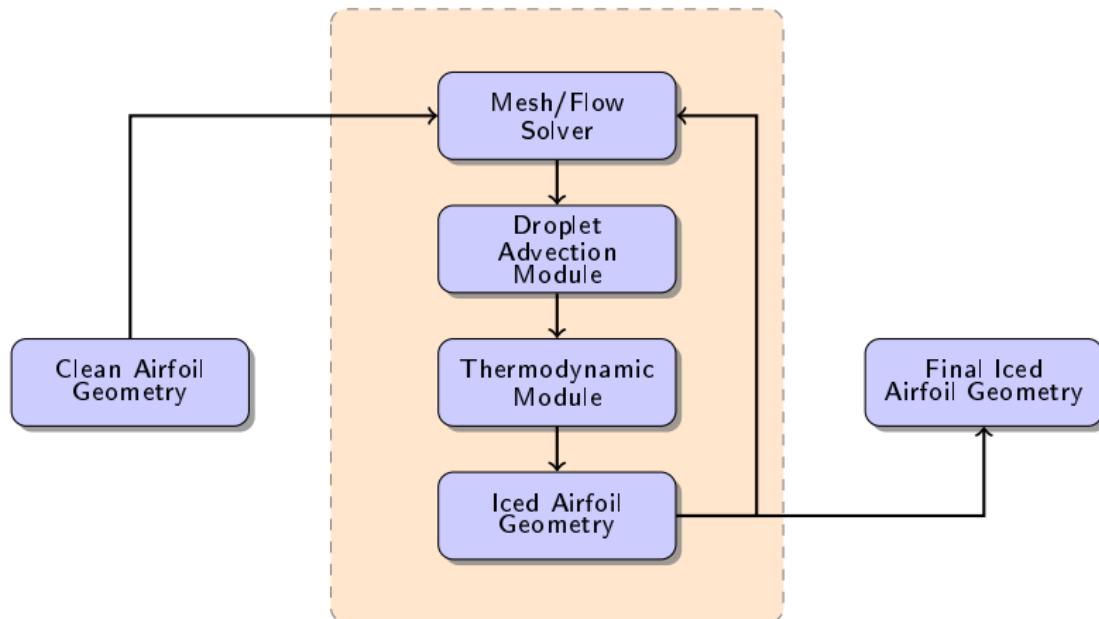
Motivation

Investigate uncertainty in the physical process of icing

- What is the statistical effect of uncertainty in physical parameters?
 - Free-stream temperature
 - Angle of attack
 - Convective heat transfer
 - Droplet diameter distribution
 - Accretion time
- Previous two approaches show how direct perturbations of the shape affect the aerodynamics
- This approach shows how perturbations of the physics affect shape (and aerodynamics)



Airfoil Icing Code Flowchart





Droplet Advection

Advection Equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$m \frac{d\mathbf{v}}{dt} = \frac{1}{2} \rho_g C_D \pi r^2 ||\mathbf{v}_g - \mathbf{v}|| (\mathbf{v}_g - \mathbf{v}) + m\mathbf{g}$$

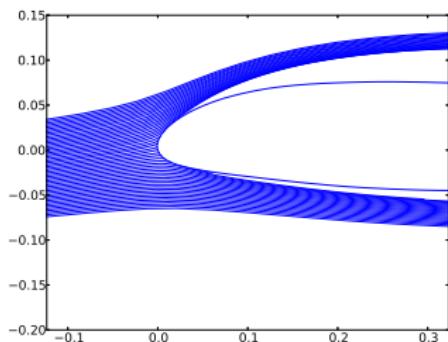


Figure: $R = 10\mu\text{m}$

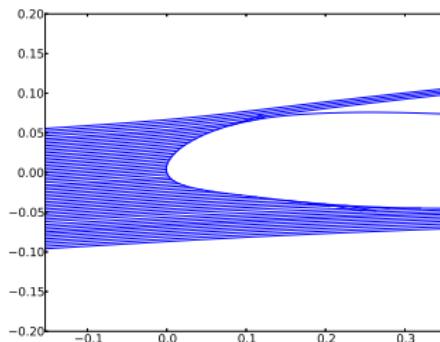


Figure: $R = 100\mu\text{m}$



Thermodynamics

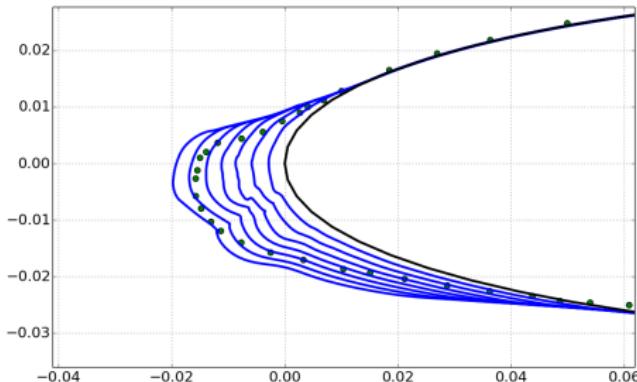
Conservation Equations:

$$\begin{aligned}\rho_w \left\{ \frac{\partial h_f}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f) \right\} &= \dot{m}_{imp} - \dot{m}_{evap} - \dot{m}_{ice} \\ \rho_w \left\{ \frac{\partial(h_f c_w T)}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f c_w T) \right\} &= \left[c_w T_d + \frac{u_d^2}{2} \right] \dot{m}_{imp} \\ &\quad - L_{evap} \dot{m}_{evap} \\ &\quad + (L_{fus} + c_{ice} T) \dot{m}_{ice} \\ &\quad + c_H (T_{Rec} - T)\end{aligned}$$

- Mass
 - Enters through impinging droplets
 - Exits via evaporation/sublimation and freezing
- Energy
 - Enters through impinging droplets, freezing of ice
 - Exits via evaporation/sublimation, radiation, convection
- Solution procedure: explicit marching, finite volume discretization with upwinded derivatives



Preliminary Intermediate Results: Ice Shapes



- NACA0012, $\alpha = 4^\circ$, $T_\infty = 256K$, $U_\infty = 103 \text{ m/s}$, MVD = $20 \mu\text{m}$, LWC = 0.55 g/m^3 , Re = 4.14 million, $\Delta T = 7 \text{ min}$
- Low temperatures: convective heat transfer high enough to freeze all incoming droplets instantly (rime)



Work In-Progress

- Verify icing calculations against published results
- Perform UQ studies, investigate sensitivity to physical parameters
 - Temperature, convective heat transfer coefficient, Reynolds number, MVD, LWC, angle of attack, etc.



Conclusions

Problems:

- Wing icing deteriorates icing aerodynamics, danger to safe flight
- Ice shapes are diverse and complex
- Not clear what the exact aerodynamic effects of different shapes are

Solutions:

- Demonstrated three separate approaches to quantifying the effects of icing uncertainty on airfoil aerodynamics
 - Heuristic approach
 - Perturb template shape with a few scaling parameters
 - Data-driven approach
 - Build model of ice shape variation from database, study the model
 - Computational approach
 - Build computational ice accretion code, perturb physical parameters